MODAL REDUCTION OF VISCOELASTIC BEAM COSIDERING HIGHER ORDER SYSTEM

A thesis submitted in partial fulfillment of the

Requirements for the degree of

Master of Technology

in

Machine Design and Analysis

By

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CERTIFICATE

This is to certify that the summer project report entitled, " ***MODAL REDUCTION OF VISCOELASTIC BEAM COSIDERING HIGHER ORDER SYSTEM***" submitted by ***Mr. K Praveen(213ME1374)***, in partial fulfillment for the requirements for the award of **Masters of Technology** in **Mechanical Engineering** with specialization of "***Machine Design & Analysis***" from National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the report *has not been submitted to any other University/Institute for the award of any Degree or Diploma.*

Dr. (Prof.) H.P.ROY
Dept. of Mechanical Engineering
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India

Place:
Date:

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I am grateful to my supervisor Dr. Haraprasad Roy, whose valuable advice, interest and patience made this work a truly rewarding experience on so many levels. I am also thankful to my friends and colleagues for standing by me during the past difficult times. Particularly, I am indebted to Mr. Saurabh Chandraker and Mr. Saurabh Sarma for their utterly selfless help. As for Samarth mishra, Naveen Tatapudi, Rasula venky, Satish Adireddy, Himanshu,Vinay reddy : You were there for me when really needed and I am yours forever.

K Praveen

ABSTRACT

Most of the finite element models are very large in size with thousands of degrees of freedom Modal reduction techniques are applied to reduce the size of large finite element models to give faster computations of the required dynamic characteristics. There are many model reduction techniques which are used to model the system with small amount of errors. A popular method of model reduction is balanced reduction method. The present work consists of the study of full modal and reduced modal vibrational dynamic characteristics of a simply supported beam considering higher order with viscoelastic materialistic nature which is based on Euler-Bernoulli beam theory. The typical characteristics of a viscoelastic material depends on temperature, strain, dynamic excitation, etc.., The main objective of this work is to do modal reduction of the higher order viscoelastic material using balanced truncation (or) reduction (or) realization and to compare the results with full modal. For this, a fourth order equation of motion is developed for a simply supported beam with viscoelastic linear damping for different temperatures. Frequency, time and transient responses of the beam are plotted by using MATLAB software. Model reduction is done via balanced truncation.

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CHAPTER 1

INTRODUCTION

1.1 VISCOELASTICITY

There are some materials like polymers, plastics, biological materials and metals at high temperatures which when subjected to loading and unloading exhibits gradual deformation and recovery. The response of such materials depends upon how quickly load is applied and how it is removed and the extent of deformation depends on the rate at which the loading is applied, this time-dependent behavior of a material is called viscoelasticity.

From [1] Viscoelasticity is a combination of two words viscosity and elasticity. Viscosity is a property of fluid which is defined as the resistance offered by fluid to flow whereas Elasticity, is a material property. So a viscoelastic material is a material which possesses both fluid and solid properties. For linear viscoelastic materials, the constitutive relationship between stress and strain can be expressed as:

$$\sigma = \sigma(\varepsilon, \dot{\varepsilon}) \qquad (1)$$

From equation (1), viscoelasticity is not only a function of strain ε but also its rate the time-dependent material behavior is called viscoelasticity. Viscoelasticity is made up of two words viscosity and elasticity. Viscosity is a fluid property and is a measure of resistance to flow. Elasticity, on the other hand, is a solid material property. Therefore, a viscoelastic material is one that possesses both fluid and solid properties. The general stress strain relationship does not include the time dependent behavior but some materials depend on the time based stress strain relationship.

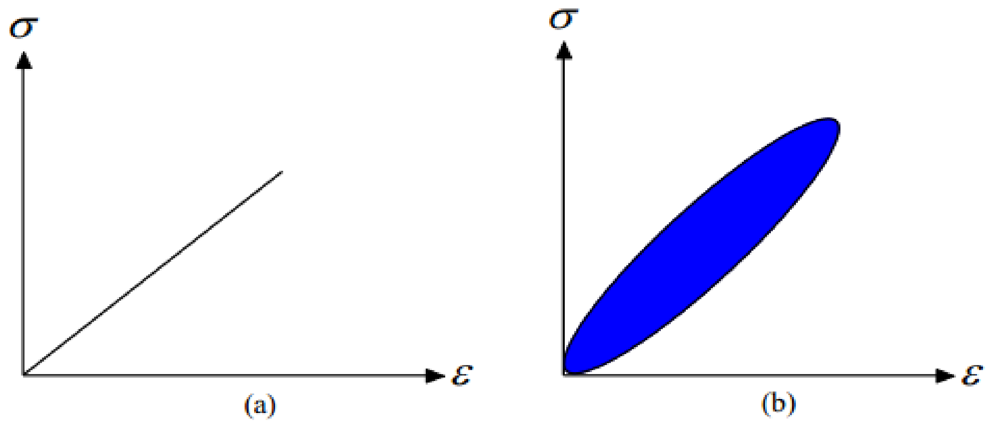

Figure1.1: Stress-Strain Graph for Elastic and Viscoelastic Material

1.2 MODEL REDUCTION

In design of systems such as micro-electro-mechanical (MEMS) the equations that define the system are written in partial differential equations to simulate such systems finite element model discretization is required which provides ordinary differential. This discretization results in large scale systems of ordinary differential equations. As it is very difficult to compute such large sized systems. So model order reduction is required. Model reduction techniques are very useful for efficient dynamic analysis of large finite element problems. The mass and stiffness matrices from finite element formulation contains thousands of degrees of freedom they are to be reduced to a smaller sets about hundreds of degrees of freedom as only some of the degrees of freedom plays role in the dynamics of the system while other plays less significant roles, so it is desired to reduce such degrees of freedom so that complexity of calculation and for analysis is reduced.

Several model reduction techniques are there some of them are

Guyan/Irons Condensation

Improved Reduced System (IRS)

System Equivalent Reduction Expansion Process (SEREP)

Dc Gain ranking method

Balanced Reduction method

1.2.1 GUYAN REDUCTION METHOD (DYNAMIC CONDENSATION)

Guyan developed the standard method of model reduction in which the mass and stiffness matrices are rearranged as the independent or master degrees of freedom and slave or dependent degrees of freedom. Typically in dynamic condensation displacement degrees of freedom of higher mass nodes are taken as master degrees of freedom and rotational degrees of freedom with higher mass moment of inertial nodes and displacements of lower inertial nodes and lower mass moment of inertial nodes are taken as slave degrees of freedom. Dynamic condensation is similar to static condensation but it contains mass or inertial elements. In general for dynamic condensation mass matrix is consistent mass matrix taken from classical paper of archer in 1963[8]. Guyan reduction introduces errors as we remove some contributions from lower inertial nodes. The magnitude of errors depends on the choices of degrees of freedom that are to be reduced.

1.2.2 IMPROVED REDUCED SYSTEM

In this improved reduction system [3] the inertial terms present with the slave degrees of freedom are taken into consideration which were neglected in guyan reduction method. This method was developed by O'Callahan he considered an extra term in the transformation matrix of the static reduction technique to make allowance of the inertial terms. This extra term makes the modal vectors to be approximated more accurately in the full model. Two methods extended the IRS method first by using the transformation from dynamic reduction and second by using the iterative method.

1.2.3 SYSTEM EQUIVALENT REDUCTION EXPANSION PROCESS (SEREP)

The system equivalent reduced expansion process depends on the subset of modes in the full system. In SEREP system [2] there will be more master degrees of freedom than the modes of interest .The transformation function has more terms, extra terms of master degrees of freedom are added. If the master degrees of freedom are equal to modes of interest then the transformation matrix will be simplified. As inertial terms are added to the master degrees of freedom this model is more accurate than Guyan and IRS systems. This system partitions the system into measurable and unmeasurable modes. The transformation is obtained by multiplying the equations with generalized pseudo inverse. At low frequencies the reduced system has eigenvalues equal to the full modal system's eigenvalues.

1.2.4 DC GAIN RANKING

In DC Gain ranking method [19] for any mode, if the degree of freedom associated with the applied force has a zero value, then the force applied at that degree of freedom cannot excite that mode, so the dc and peak gains will also be zero. If the mode cannot be excited, then it has no effect on the frequency response and can be eliminated.

1.2.5 BALANCED REDUCTION

The concepts of controllability and observability in the control community can be used to rank the modes with some ambiguity involved.in general the controllability in a mode cannot be related to observability and so as observability of a mode cannot be related to controllability.

The balanced reduction technique simultaneously takes into account of both controllability and observability in its rankings and overcomes the uncertainty involved in using either controllability or observability alone.

The balanced method provides slightly better impulse results than the dc gain method, for models with same number of retained states. For frequency response the balanced reduction method fits one additional mode over that of dc gain method.

1.3 STABILITY

The physical parameters m, c and k are generally positive quantities but in some situations the expressions contains one or more negative coefficients. Then the system behaves well and can be treated as stable

1.4 ANELASTIC DISPLACEMENT FIELD METHODS

The need to produce finite element method [7] that are capable of producing dynamic characteristics of a structure or beam made lesieutre developed an independent means of augmenting finite element methods containing damped properties found from material loss factor curves. Lesieutre method uses a first order state space method called Anelastic displacement fields (ADF) methods

CHAPTER 2

LITERATURE SURVEY

George A lesieutre and kiran govindaswamy [1] in their finite element modelling of frequency dependent and temperature dependent dynamic behavior of viscoelastic materials paper showed that material behavior is dependent strongly for temperature and frequency changes.

C. H. Park, J Inman and M J Lam [2] in their paper of model reduction of viscoelastic finite element models discussed that the GHM (Golla Hughes McTavish) method with model reduction techniques examines the behavior of the material for the frequency response when the properties of viscoelastic dampng are added to the finite element model for a lower order model which obtained by reducing the higher order model of the original system.

From the paper of the convergence of iterated Irs method M I Friswell [3] showed that two approaches in IRS model the first is transformation matrix formed from dynamic condensed system and iterated IRS method and the later method that is iterated IRS method converges the results obtained from iterated improved reduced system (IRS) equal to the results obtained in system equivalent reduced expansion process (SEREP).

Dale F Enns [4] in his model reduction with balanced realizations, the error bound for reduced order with balanced truncation is derived. The importance of the infinity norm and frequency weighted model reduction was discussed.

Michael I Friswell [5] in his paper of the reduced order models of structures with viscoelastic components discussed that in order to calculate the transient response of a viscoelastic structure, a frequency dependent damped model is introduced with extra dissipation coordinates. For a range of frequencies these coordinates requires a curve fit to material loss factor data. The introduction of these extra degrees of freedom makes the model very complex so he used balanced reduction method to the complexity of the system.

Fei xe.et al [6] in their paper of correlation analysis of PCB and comparison of test analysis model reduction methods presented that for choosing the best excitation point a method based on the modified modal participation approach. By using a printed circuit board a comparision is made between modified modal participation method and modal participation method where the results proved the modified method produces better results.

Michael I Friswell and D J Inman [7] in their paper of finite element models with viscoelastic damping discussed that the frequency dependence of a viscoelastic model by introducing new dissipating external coordinates or internal variables for the calculation of transient response for a structure with viscoelastic damping nature but this method makes the controller design intensive computationally. They examined the methods of reducing the full modal through eigenvalue truncation and balanced realization.an iterative method is introduced to calculate full Eigen system calculation.

Bruce C Moore [8] in his paper on principal component analysis in linear systems: controllability, observability, and model reduction discussed that principal control analysis along with an algorithm for calculating singular values of a matrix can become a very powerful technique to deal with the structural instability of dynamical systems . kalman's minimal realization technique is remodelled to calculate the response for the given input signals and application of signal analysis to controllability and observability provides a new coordinate system which is balanced and has special properties.

A C Antoulas et.al [9] in their publication A survey on model reduction methods for large scale systems given an overview of model reduction techniques and their procedures and distinguished two categories of models based on singular value decomposition method and moment matching method and found the error of approximation in SVD method gives better results globally and moment matching method gives good approximations locally.

Thilo penzl [10] in his paper on algorithms for model reduction of large dynamical systems described three algorithms for model reduction of large scale linear time invariant dynamical systems. They depends on the rank given for controllability and observability grammians which are computed by ADI based low iterative rank methods. Out of the three algorithms two were balanced realization techniques related to schur method and square root method. The third is heuristic method based on balancing free technique.

Carolyn L Beck et.al [11] in his model reduction of multi-dimensional and uncertain systems discussed that linear fractional transformation for scalar uncertain structures involves the generalization of balanced realization techniques with conformed errors based on linear matrix inequalities which generalize lyapunov equations. These can be applied uncertainty simplification which was shown in his paper.

David C Hyland et.al.. [12] in their paper of the optimal projection equations for model reduction and the relationships among the methods of Wilson, skelton and moore first order necessary conditions for quadratically optimal reduced order model of linear time invariant systems are derived in the form of a pair of modified lyapunov equations coupled by an oblique projection which determines optimal reduced order model. This form of necessary conditions simplifies results of Wilson and clearly demonstrates the quadratic extremality and non-optimality of the balancing method of moore. The possible existence of multiple solutions of the optimal projection equations is demonstrated and a relaxation-type algorithm is proposed for computing these local extrema. A component-cost analysis of the model-error criterion similar to the approach of Skelton.

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K. Vincekza Fernand [13] in his paper on Singular perturbational model reduction of balanced systems he balanced representations of linear systems due to Moore are shown to be natural and convenient mediums for singular perturbational model reduction.

Liqian Zhang et.al [14] in their papers $\boldsymbol{H}_{\infty}$ model reduction of markovian jump on linear systems model reduction problem for linear systems that possess randomly jumping parameters is studied. The development includes both the continuous and discrete cases. It is shown that the reduced order models exist if a set of matrix inequalities is feasible. An e8ective iterative algorithm involving linear matrix inequalities is suggested to solve the matrix inequalities characterizing the model reduction solutions. Using the numerical solutions ofthe matrix inequalities, the reduced order models can be obtained. An example is given to illustrate the proposed model reduction method.

CHAPTER 3

FINITE ELEMENT METHOD:

Many physical phenomenon are described by using partial differential equations. Solving these equations for complex geometries by analytical methods is almost impossible. So numerical methods [7] are developed in order to solve these equations so methods such as finite element method, finite difference methods are developed. The finite element method is a numerical method which solves the partial differential equations approximately. The approximate solutions becomes exact.

When the geometry is divided into infinite elements and geometry must be defined with complete set of polynomials.

The main concept of Finite Element Method [17] is to make the complex geometric model into simple small elements as finite elements. These elements which were divided forms the important part in the analysis. The elements which were divided can be in different forms. The division of the domain into elements is called meshing. The forces and moments that are applied at certain elements or transferred from one element to the other elements are represented by degrees of freedom. The locations at which these degrees of freedom are defined are known as nodes.

The general finite element method consists of the following five step procedure:

Step1: subdivision of the domain to number of finite elements, now a days it is performed by automatic mesh generators. If done manually the major task is to select the appropriate number of elements and variation of the size of the elements and shape of the elements primary variable functions such as displacement function by using nodal values with in each element. The relationships among the forces and their effects are important in obtaining the equations for each individual elements then, element stiffness and mass matrices are developed which is the key concept in finite element method. The following are some methods for the formulation of element stiffness matrix:

3.1 DIRECT EQUILIBRIUM METHOD

This method is usually used for one dimensional or line elements. Using force equilibrium conditions for an element and relating nodal forces and nodal displacements the stiffness matrix and other related equations are derived.

Work or Energy Methods: In this method the principles of minimum potential energy and principle of virtual work are used to derive the stiffness matrix. Generally principle of minimum potential energy is used for elastic materials and principle of virtual work can be adopted to any material. This method is generally used for extraction of stiffness matrix for 2D elements.

3.2 METHOD OF WEIGHTED RESIDUALS

In this method a trial function is assumed which are defined over the entire domain. Applying method of weighted residuals contains two steps:

In first step we assume the general functional behavior of dependent functional variable to satisfy the given differential equations and given boundary conditions. By substituting this approximate function into the differential equation produces some error this error is the residual. This residual is to be vanished or error is to be minimized.

The second step is to solve the resultant came in the first step thereby specialize the general functional form to a particular functional form which then approximates the solution to the equation.

Step3: Assembling the equations obtained for each elements for the whole physical problem and applying the boundary conditions. In this step the equations that were developed for elements are combined to form a global equations for the full physical domain. After developing global matrices boundary conditions and external forces or loads are applied.

Step4: Solving the equations, in this step the equations are solved for primary and secondary variables by using the displacement equations developed in the process

Step5: Post processing of the solved equations for the quantities of interests like displacements stresses, strains, etc…, and obtaining the visual effects of these equations. It is important to interpret the results as it will show the critical points if present for better results and to avoid the failure.

3.3 APPLICATIONS OF FINITE ELEMENT ANALYSIS

There are two types of problems in finite element method they are Structural Problem and non-structural

Structural problem:

1) Stress Analysis
    - truss & frame analysis
    - stress concentrated problem
2) Buckling problem
3) Vibration Analysis
4) Impact Problem

Non-structural problem:

1) Heat Transfer
2) Fluid Mechanics
3) Electric or Magnetic Potential

Chapter 4

ANALYSIS OF STATE SPACE MODEL

4.1 STATE SPACE REPRESENTATION

The state space representation can be thought as a partial reduction of equation to a set of simultaneous differential equations. The state variables of the system are not unique so that we can apply the operations performed in linear algebra [. Operations performed on a set of state variables can lead to a set of new state variables.

The higher order equation which was developed earlier is difficult to solve using ordinary mathematical techniques such as differential equations and numerical methods so in order to solve this higher order equation we have to use state space approach.

State space representation is used by control system engineers to represent a complex system which is mathematically described by an n^{th} order differential equation. The state space model which we are using is of linear time invariant type.

It is represented as,

$$\dot{x} = Ax + Bu \qquad (4.1)$$

This is a first order differential equation is known as the state equation. The matrix **A** is called as system matrix which is an n by n square matrix and **B** is an input matrixof the order n by r. The state differential equation relates the rate of change of the state of the system to the state of the system and the input signals.

For dynamical systems the state of a system [16] can be described by a set of variables called state variables.the state variables are the variables that determine the behavior of a system when the present state of the system and the excitation signals are known

$$[x]=[x_1(t) \quad x_2(t) \quad \dots \quad x_n(t)]$$

Where $[x]$ is a vector of state variables.

The outputs of a linear system can be related to the state variables and the input signals by an algebraic output equation

$$y = C\,x + D\,u \qquad (4.2)$$

Where, **y** is the set of output signals.

The state-space representation is comprised of the state variable differential equation and the output equation.as the two equations share the state variables The state equations are coupled to each other. The state equations must be solved before the output equations are solved. Solution of state equations requires a numerical method for solving simultaneous differential equations. So, we will use Runge-Kutta method to solve these equations. The Runge-Kutta method is available in MATLAB and the solution for output is equation is of algebraic form in terms of state variable and input variable.

4.2 STABILITY OF A LINEAR SYSTEM

Consider a general form of the nonlinear system [16]

$$\dot{x} = f(x,t) \qquad (4.3)$$

$$x(t_0) = x_0 \qquad (4.4)$$

Where x is a vector and $f(x,t)$ represents a general nonlinear function.

The above system is a stable in the lyapunov sense with respect to the equilibrium state if for a given value $\varepsilon > 0$,there exists a number $\delta(\varepsilon, t_0) > 0$ for which the $\|x(t)\| < \varepsilon$ for all $t > t_0$ and $\|x(t)\| < \delta$

The above condition implies that magnitude of $x(t)$ remains within a finite small value in the presence of small initial perturbation. This definition includes undamped pure oscillatory motions.

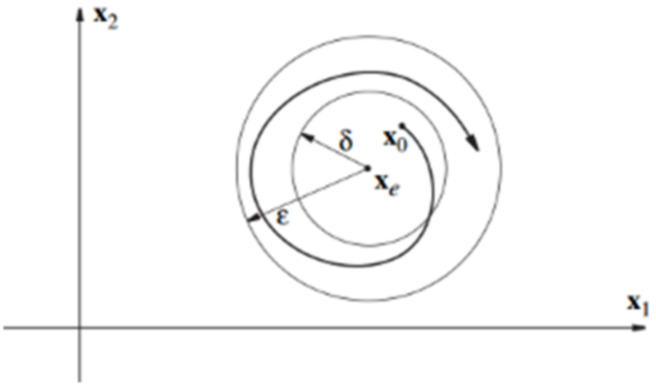

Figure 4.1: showing stability

If the system is started somewhere at x_0 within the disc with radius δ then the trajectory will remain within the disc with radius ε at all times.

4.3 ASYMPTOTIC STABILITY

The system is asymptotically stable if it satisfies the stability and

$$\lim_{t \to \infty} \|x(t)\| = 0$$

The asymptotic stability implies that the state vector converges to the equilibrium point which is generally assumed to zero at steady state.

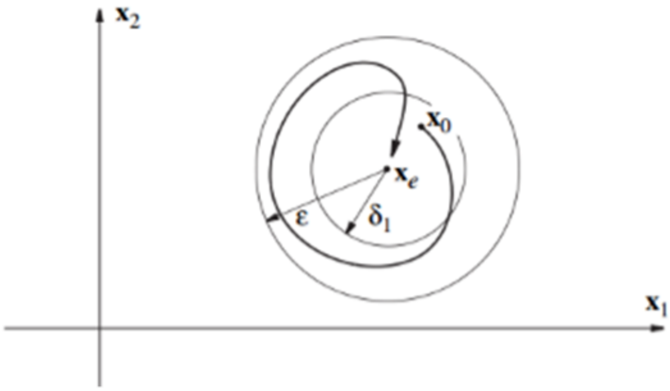

Figure 4.2: showing asymptotic stability

4.4 LYAPUNOV SECOND STABILITY THEORY

Non-negative energy function is used for stability in this theory for a system. The energy function is used a measurement for stability if the energy of the system decreases then the system is stable towards the equilibrium point. Therefore, the system energy is minimum toward the equilibrium point.

Theorem:

Let $V(x)$ be the lyaounov or energy function. For all values of 'x' the system is stable if $V(x) > 0$ and $\dot{V}(x) \leq 0$. If the rate of $V(x)$ is < zero then the system is asymptotically stable.

If the lyapunov function is not available then we cannot draw any conclusions on the stability of the system. This is the drawback in lyapunov's approach there is no systemic approach is provided to calculate lyapunov function instead we need to assume the function.

The study of controllability and observability is an important aspect for studying the internal structure of the system.

4.5 CONTROLLABILITY

The system is controllable if there exists a control input and time t_f by which an arbitrary $\{x(t_f)\}$ can be reached from with $t_0 < t \le t_f$.

In a mathematical theorem from [15], it is stated as:

$$[G_o(t_f, t_o)] = \int_{t_0}^{t_f} [e^{[A](t_f-\tau)}[B][B]^T e^{[A]^T(t_f-\tau)} d\tau \tag{4.5}$$

The controllability condition for a linear system is given as:

$$\dot{x} = \{A\}\{x\} + \{B\}\{u\} \tag{4.6}$$

And the condition required is

$$[G_c(\xi)] = \int_{t_0}^{t_f} e^{[A](t_f-\tau)}[B][B]^T e^{[A]^T(t_f-\tau)} d\tau \tag{4.7}$$

$$[A][G_c] + [G_c][A]^T + [B][B]^T \tag{4.8}$$

$$[P] = \left[[B],[A][B],[A]^2[B],\ldots,[A]^{n-1}[B]\right] \tag{4.9}$$

This condition should be positive definite that is for the matrix $[G_c(t_f, t_0)]$ inverse should exist. the matrix $[G_c(t_f, t_0)]$ is called controllability grammian.

Without loss of generality, we take $t_f = \infty$. In this state the controllability is tested throughout the steady state. Therefore the controllability grammian turns into

$$[G_c(\xi)] = \int_{0}^{\infty} e^{[A](\xi)}[B][B]^T e^{[A]^T(\xi)} d\tau \tag{4.10}$$

Where $\xi = t_f - \tau$

The matrix $[G_c(\xi)]$ will satisfy

$$[A][G_c]+[G_c][A]^T+[B][B]^T \tag{4.11}$$

Which is called as lyapunov equation

There is an alternate form of controllability condition. That is, the rank of a controllability matrix

$$[P]=\left[[B],[A][B],[A]^2[B],...,[A]^{n-1}[B]\right] \tag{4.12}$$

Should be of same order of that of the system.

4.6 OBSERVABILITY

Another important issue in modern control design is observability [15]. The observability of a dynamic system represents the ability of reconstructing all state variables using a finite number of sensor outputs. In the majority of modern control system designs and analyses, the number of sensors is less than that of the state variables due to actual constraints. Also, it will be a significant advantage if we can estimate all stable variables using only a limited number of sensors.

The observability is a primary requirement estimating the state variables out of direct sensor output. Mathematical description of the observability condition is similar to that of controllability.

Definition of observability

A system is stable if and only if any state $\{x(t)\}$ can be determined by using a finite output $\{y(\tau)\}$, for $t<\tau\leq T$

The mathematical theorem is stated as:

THEOREM

A system is observable if and only if the matrix [15]

$$[G_o(T,t)] = \int_t^T [\phi(\tau,t)^T [C]^T [C][\phi(\tau,t) d\tau \qquad (4.13)$$

Is positive definite .this matrix is called the observability grammian

Similar to controllability if we take $T = \infty$ then it follows

$$[G_o] = \int_t^T [\phi(\xi)^T [C]^T [C][\phi(\xi) d\xi \qquad (4.14)$$

Also, $[G_o(\xi)]$ turns out to satisfy the lyapunov equation

$$[A][G_o] + [G_o][A]^T + [C][C]^T = 0 \qquad (4.15)$$

In an alternative way, if the system is observable if and only if the observability matrix

$$[Q] = \left[[C]^T, [A]^T [C]^T, [A^2]^T [C]^T, \ldots, [A^{n-1}]^T [C]^T \right]^T \qquad (4.16)$$

Has rank n, the order of the system.

Chapter 5

METHODOLOGY

5.1 FORMULATION OF EQUATION OF MOTION

The equation of motion of the system which is obtained by assembling various elements into one global matrix is:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f\} \qquad (5.1)$$

Where, $[M]$ $[C]$ and $[K]$ are global mass, damping and stiffness matrices respectively and $\{f\}$ is the forcing function.

If we could express the damping matrix as a combination of mass and stiffness by considering linearity we can increase the order of the variables, here we consider a second order linear equation for damping matrix and we can express it as a combination of mass and stiffness matrices.

$$m\ddot{x}+kx=0 \qquad (5.2)$$

if $'k'$ *operator* is assumed as, $k()=\dfrac{a_1+a_2D+a_3D^2}{b_1+b_2D+b_3D^2}$

$$mD^2x+\frac{a_1+a_2D+a_3D^2}{b_1+b_2D+b_3D^2}kx=0 \qquad (5.3)$$

where,'D' is differential operator

$$m(b_1+b_2D+b_3D^2)D^2x+k(a_1+a_2D+a_3D^2)x=0 \qquad (5.4)$$

$$mb_1D^2x+mb_2D^3x+mb_3D^4x+ka_1x+ka_2Dx+ka_3D^2x=0 \qquad (5.5)$$

arranging the equation,we get

$$mb_3D^4x+mb_2D^3x+(mb_1+ka_3)D^2x+ka_2Dx+ka_1x=0 \qquad (5.6)$$

assuming ,

$$A_1=mb_3, A_2=mb_2, A_3=mb_1+ka_3, A_4=ka_2, A_5=ka_1$$

$$A_1\ddddot{x}+A_2\dddot{x}+A_3\ddot{x}+A_4\dot{x}+A_5x=0 \qquad (5.7)$$

5.2 INTERNAL BALANCING METHOD

This is important and basis for model reduction in control theory [12]. it works with systems having damping and is accurate but it doesn't provide connection from physical domain .the equation of motion which was defined for the physical system in higher order differential equations must be converted into state space form. As matrix B and matrix C are directly related to the forces that were applied and the measurement they play key role in determining controllability and observability of the system. The important necessary condition of the system is matrices A B and C should be asymptotically stable, controllable and observable. The balanced reduction technique allows the states which contribute less response in the system are removed. For asymptotically stable systems the useful measure of the removal of states that do not contribute is to calculate controllability grammians and observability grammians. The controllability and observability grammians are unique, symmetric and positive definite and they satisfy the lyapunov stability theory Moore [8] has shown that there exists a coordinate system such that the controllability grammian and observability grammian becomes equal and then that system whose controllability grammian equals the observability grammian is said to be balanced. Calculating the singular values and ranking the state accordingly gives the reduced model.

Let, the transformation matrix which converts full model into reduced model is assumed as 'P' and transformation is resulted as follows:

1) We know controllability grammian 'G_c 'so, find find eigenvalues and eigenvectors for 'G_c 'and let the eigenvalues be assigned as 'λ_c' and eigenvectors as 'V_c 'then, calculate the intermediate transformation matrix assign it as ' P_1' and solve for

$$V_c^T G_c V_c = \lambda_c \qquad (5.8)$$

$$P_1 = V_c \lambda_c^{-1/2} \qquad (5.9)$$

2) Let the system matrices be defined as $(\bar{A}, \bar{B}, \bar{C})$ after the formation of intermediate transformation matrix. As, $\bar{A} = P_1^{-1} A P_1$, $\bar{B} = P_1^{-1} B$, $\bar{C} = C P_1$

3) The similar method is applied to observability matrix' G_o' also by using another intermediate transformation matrix 'P_2' and calculating the eigenvalues 'λ_o' and eigenvector 'V_o'

4) The transformation matrix is given by $P = P_1 P_2$ and the reduced system matrices are calculated.

One issue with the balanced reduction method is we lose the ability to directly identify the individual modes in the reduced system. After balanced reduction one needs to observe the system matrix to identify which modes are included, while dc and peak gain techniques retains the identities of the individual modes

Unlike SISO models, which can be easily ranked using simple dc and peak gain techniques, MIMO

Models will require the balanced reduction method because it easily handles the problem of ranking multiple inputs and outputs.

CHAPTER 6

RESULTS AND DISCUSSION

6.1 NUMERICAL PROBLEM

A Numerical problem is presented in order to understand the vibrational characteristics of full model fourth order beam with viscoelastic damping at different temperatures and by using balanced reduction reduced model of the same beam is examined and compared with the full model beam. The calculations are performed through the help of matlab software. A simply supported beam of internally damped viscoelastic nature is taken and discretised into 6 number of elements with seven nodal coordinates and two degree of freedom at each node is taken with a harmonic load at the centre. The length of the beam is taken as of 35 mm and width and depth of the beam is 0.035mm and the responses are plotted for three different temperatures and three elastic modulus with three constants in the ADF parameter.

TABLE OF ADF PARAMETER:

E	b1	b2	c1	c2
4.0032e+008	159.4867	1.1730e+004	25.1428	1.1067
2.3328e+008	160.4874	1.0461e+004	21.5882	1.1381
1.0672e+008	199.7878	1.2913e+004	20.7101	1.1867

The frequency response for the three different temperatures is

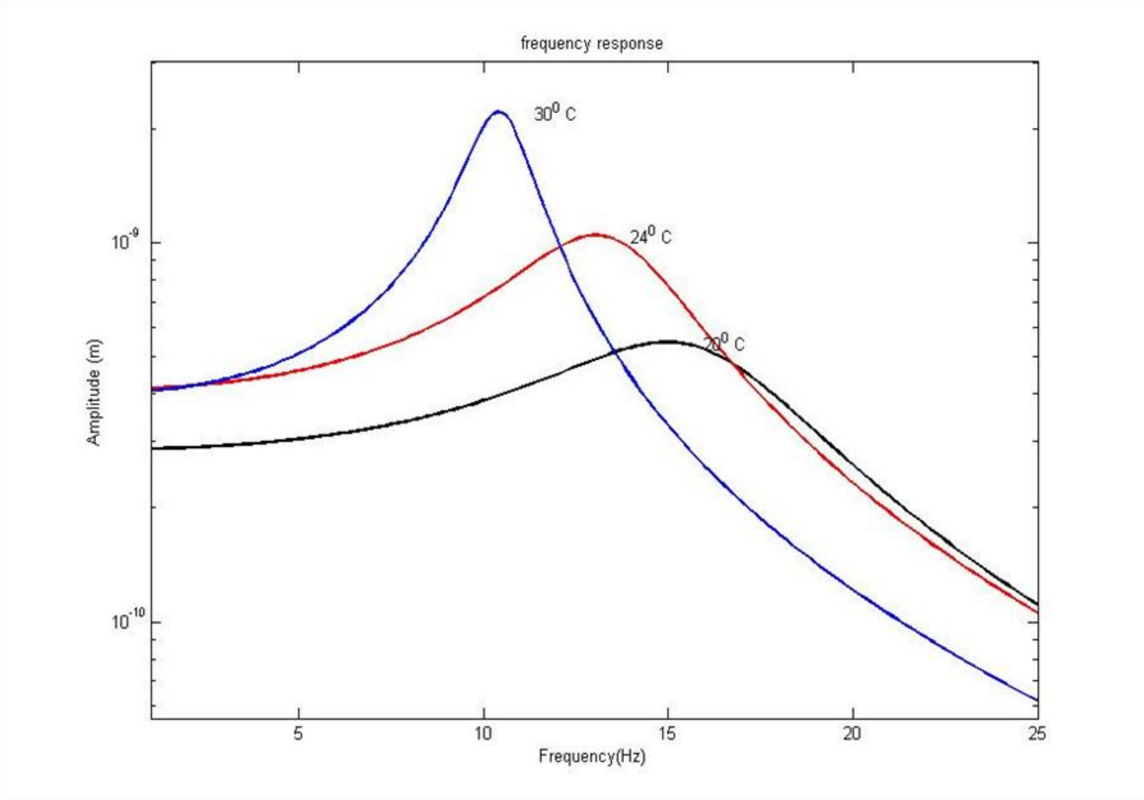

fig 6.1: frequency response at three different temperatures

The time response for the three different is plotted as

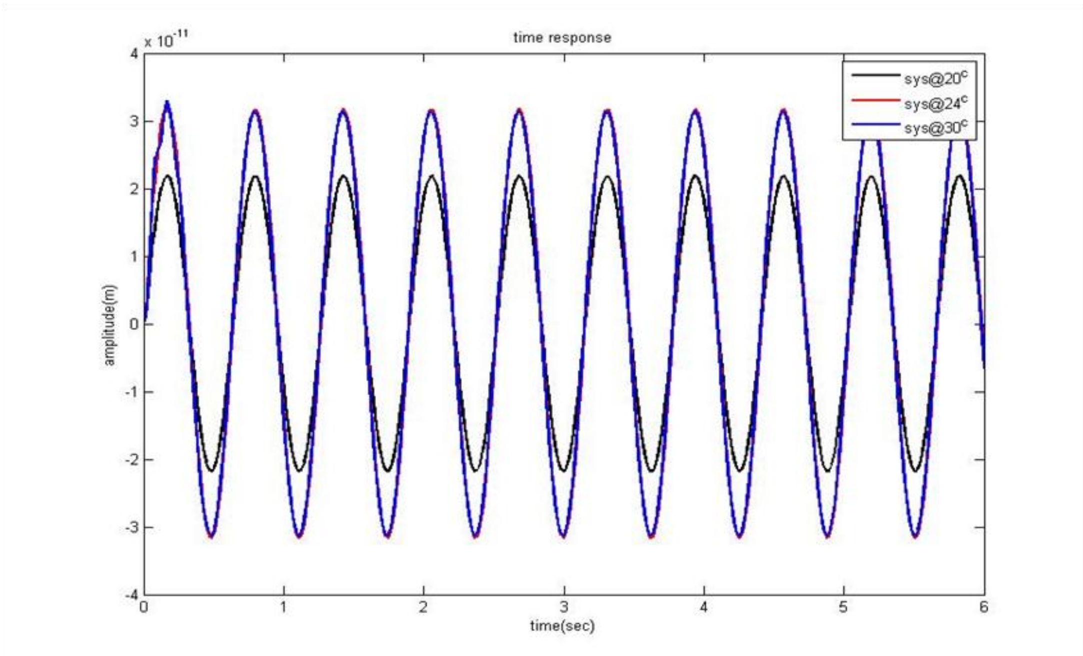

fig 6.2: time response at three different temperatures

The plots which shows the difference between full model system and reduced system at three different temperatures

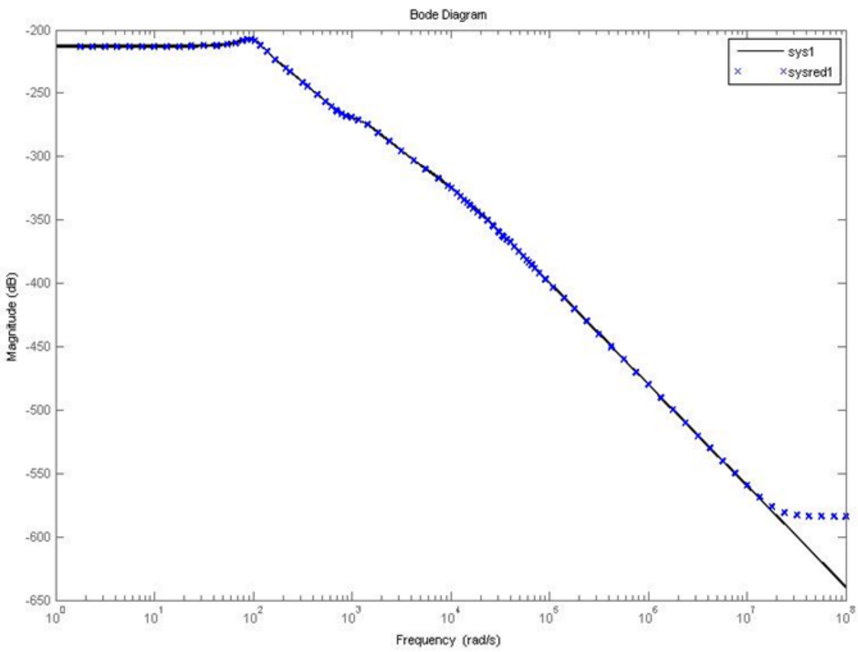

fig 6.3: frequency response at temperature 20^{c}

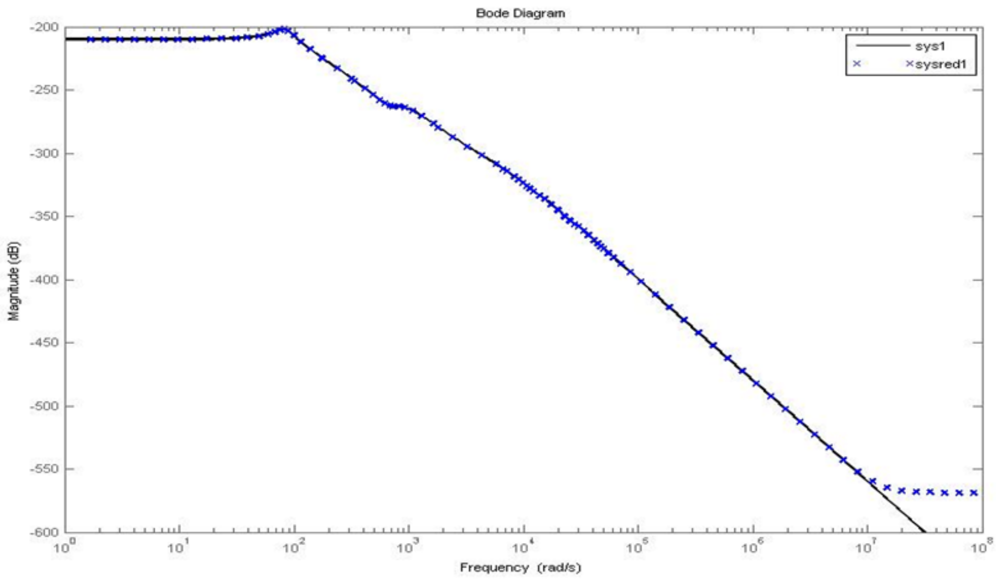

fig 6.4: frequency response at temperature 24^{c}

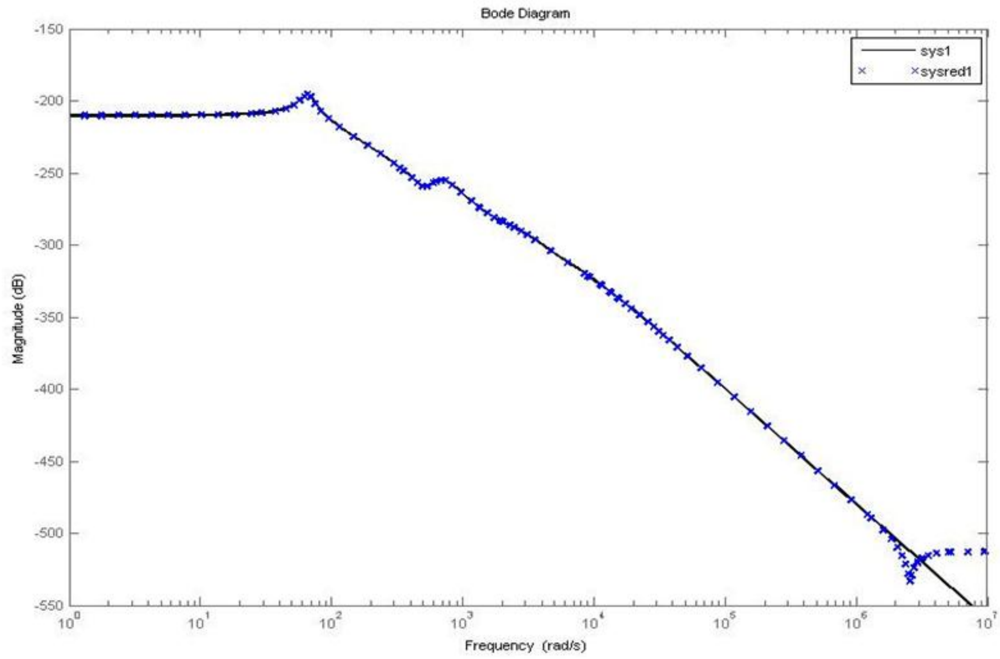

fig 6.5: frequency response at temperature 30^c

Three plots were drawn for time responses at three different temperatures

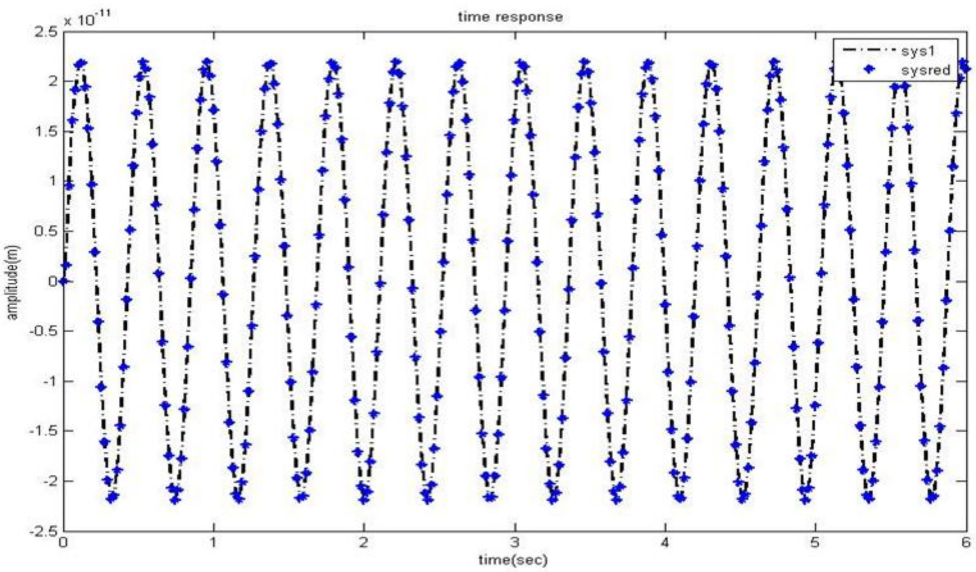

fig 6.6: time response at temperature 20^c

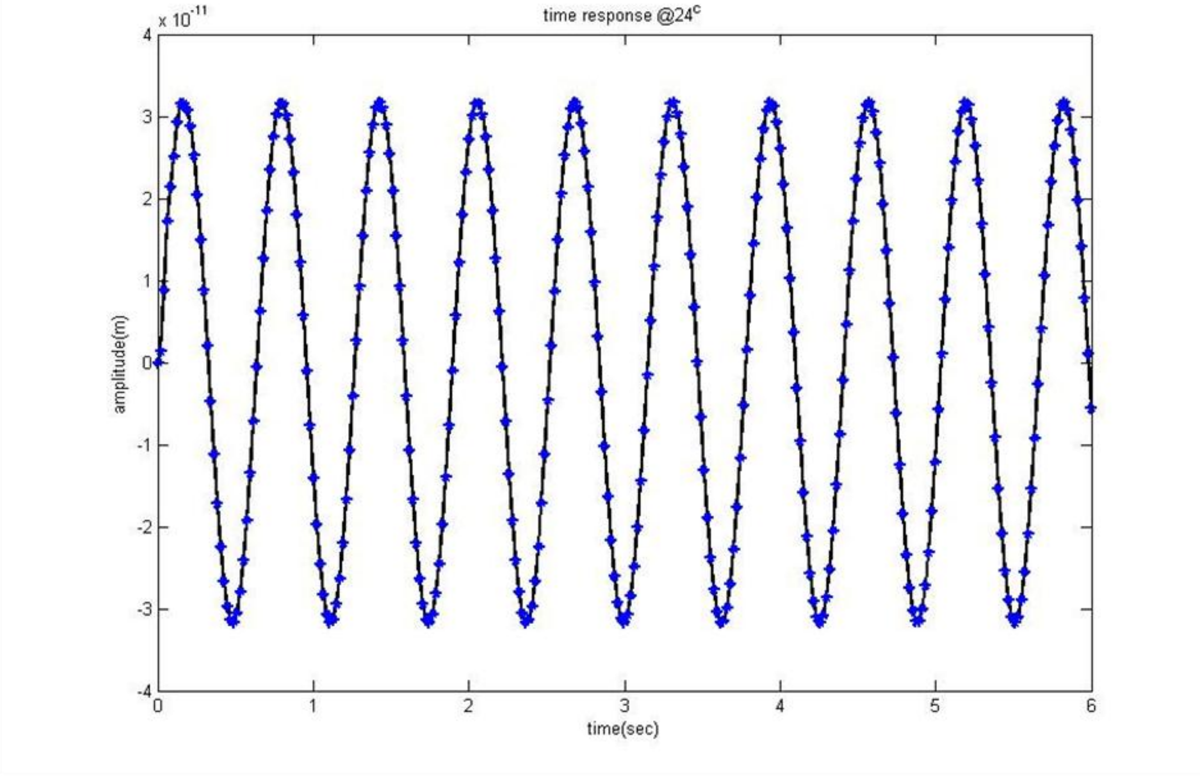

fig 6.7: time response at temperature 24^c

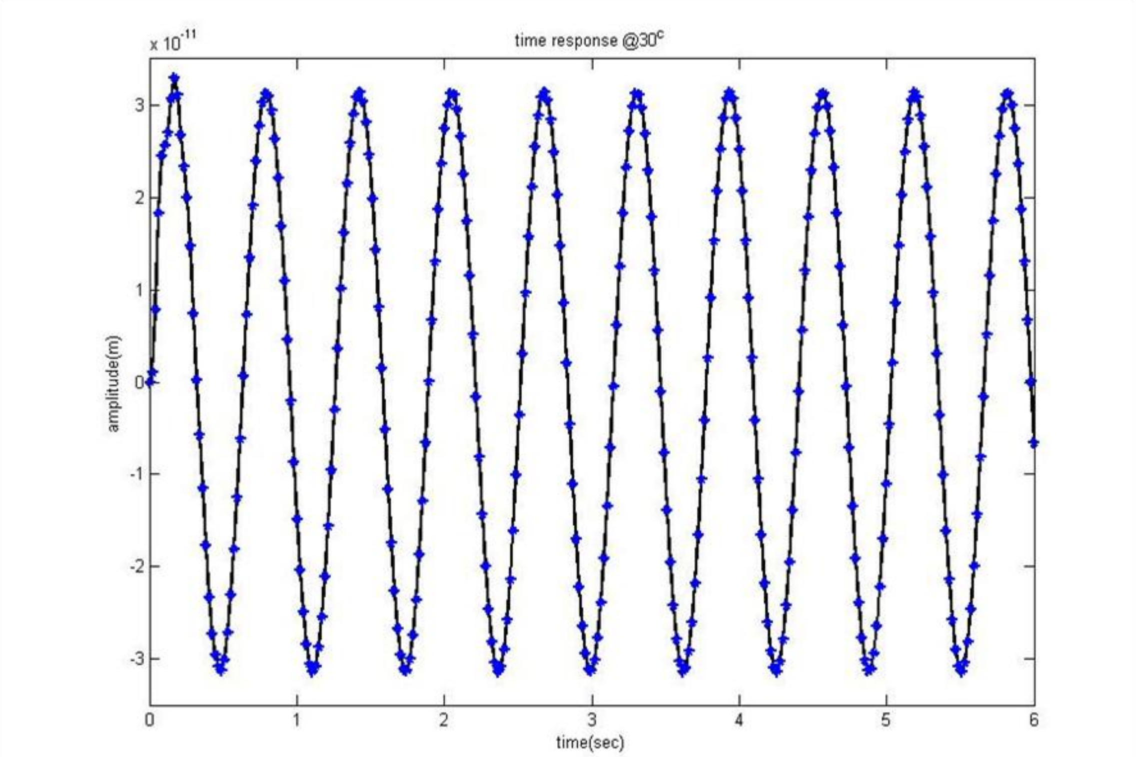

fig 6.8: time response at temperature 30^c

This is the transient response for three different temperatures given an intial condition of 0.1mm

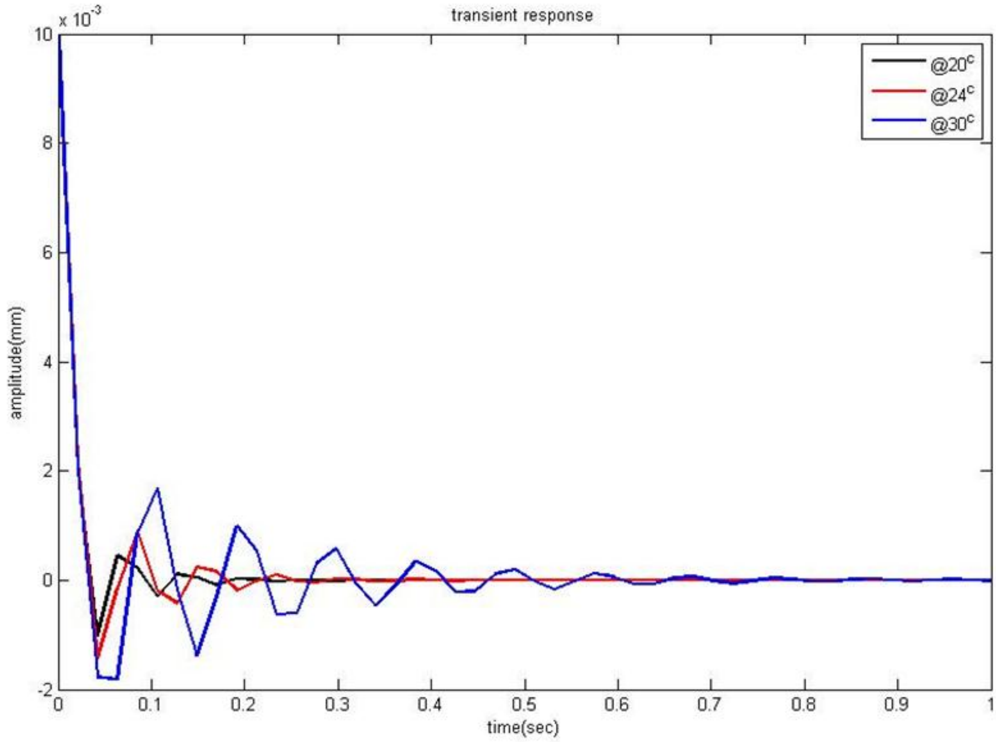

fig 6.9: transient response at different temperatures

CHAPTER 7

CONCLUSION AND FUTURE SCOPE

A method for computing the balanced reduction has been presented by using controllability grammian and observability grammians the balanced reduction method is computed from the observations made we can say that,

The balanced method provides better results when compared with other methods our numerical methods show the accuracy of the balanced reduction method.

MIMO models requires balanced reduction technique as it easily handles the problem of ranking inputs and outputs

One issue with balanced reduction is that we lose the ability to directly identify individual modes which are included in the reduced model

As many methods have disadvantage this balanced reduction method also have its own disadvantages so further research should be done to eliminate the errors that were obtained in balanced reduction method.

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